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Mixed Methods for Reduced Order Modelling of Linear Interval System

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Abstract—*This paper proposed methods to obtain a stable lower order interval model from its stable higher order interval model. In the design of control system, work has to be done with mathematical models of higher order. The analysis and synthesis of higher order model are tedious and not acceptable on economic and computational consideration. Therefor it is often in call adequately to change higher order model by lower order model. This paper proposed mixed methods for reduction of order. In mixed method two different methods are used for finding parameters of the numerator and denominator. Here Differentiation method and Routh stability criterion are used and then result is compare with reduced model obtained by Kharitonov's theorem and Routh Approximation. A numerical example has been discussed to illustrate the procedures. It is observed that model obtained from proposed method is more close to original model in comparison with model obtained from kharitonov's theorem method and it is also observed that proposed method is much easier then other methods.*

Index Terms*: Differentiation method, Interval polynomial, Reduced order modelling, Routh approximation.*

1. INTRODUCTION

For the design and analysis, approximation of higher order model to lower order make the process of simulation very easy. There has been remarkable growth in the research of reduced order modelling. There is so many methods are proposed for reduced order modelling of higher order system. From them some familiar methods are aggregation method [1], pade approximation [2], routh approximation [3] and continued fraction expansion method[4]. The drawback of above method is that it doesn't guarantee stability of the reduced order model. To overcome this problem Routh stability criterion based methods are introduced. Few of them are α-β and γ-δ Routh approximation method [5]. But these methods are for fixed coefficients only.

The systems with constant coefficients but uncertain with in finite range is comes under interval polynomial systems. The available amount of literatures exposed the study of interval polynomial for their stability and modelling analysis[7-9]. For reduction of interval system routh approximation[10], α-β routh approximation using kharitonov polynomials[13] is proposed. Differentiation method of reduction of order was introduced by [15] Gutman for fixed coefficients which is forther modified by[4] for interval polynomials.

In this paper mixed method for reduced order modelling of Linear Interval System is proposed. The numerator and denominator for reduced model is obtained by different methods. The methods used are Differentiation method and Routh stability criterion.

The highlight of paper is as follows: Problem statement is in section II; mixed method for reduction of order are in section III; section IV contains numerical example to illustrate the procedure and conclusion is in section V.

2. PROBLEM STATEMENT

Suppose the transfer function of higher order interval system be;

$$G_n(s,P,Q) = \frac{[P_0^-,P_0^+] + [P_1^-,P_1^+]s + \cdots \dots \dots + [P_{n-1}^-,P_{n-1}^+]s^{n-1}}{[Q_0^-,Q_0^+] + [Q_1^-,Q_1^+]s + \cdots \dots \dots + [Q_n^-,Q_n^+]s^n} \quad (1)$$

The reduced order model of a transfer be considered as

$$R_k(s,p,q) = \frac{[p_0^-,p_0^+] + [p_1^-,p_1^+]s + \cdots \dots \dots + [p_{k-1}^-,p_{k-1}^+]s^{k-1}}{[q_0^-,q_0^+] + [q_1^-,q_1^+]s + \cdots \dots \dots + [q_k^-,q_k^+]s^k} \quad (2)$$

The arithmetic rules for the interval polynomial have been defined[2,28[, and are written below.

Let [a,b] and [m,n] be two intervals.

Addition:

[a,b] + [m,n] = [a+m,b+n]

Subtraction:

[a,b] – [m,n] = [a-n,b-m]

Multiplication:

.

[a,b] × [m,n] = [Min(am,an,bm,bn),Max(am,an,bm,bn)]

Division:

$$\frac{[a,b]}{[m,n]} = [a,b] \times [\frac{1}{n} \times \frac{1}{m}]$$

3. PROPOSED METHODS

By the application of kharitonov theorem , four kharitonov polynomials for both numerator and denominator are obtained. The numerator polynomial and denominator polynomial are Hurwitz stable if and only if the kharitonov polynomial of the numerator and denominator are Hurwitz stable.

A. Differentiation Method

Determination of the numerator polynomial and denominator polynomial of the K^{th} order reduced model given in equation (2) by using differentiation method is as follows :

Taking the numerator and denominator array after differentiation

Table I

NUMERATOR ARRAY

s^{n-1}	$[P_{n-1}^-, P_{n-1}^+]$	$[P_{n-2}^-, P_{n-2}^+]$............$[P_1^-, P_1^+]$	P_0^-, P_0^+	
s_1^{n-1} 0	$(n-1)[P_{n-1}^-, P_{n-1}^+]$	$(n-2)[P_{n-2}^-, P_{n-2}^+]$.......$[P_1^-, P_1^+]$		
s^{n-2}	$\frac{[P_{n-2}^-, P_{n-2}^+]}{n-1}$	$\frac{2[P_{n-3}^-, P_{n-3}^+]}{n-1}$............	P_0^-, P_0^+	0

repeat he above procedure to get the numerator of reduced order s^{k-1}.

Table II

DENOMINATOR ARRAY

s^n	$[Q_n^-, Q_n^+]$	$[Q_{n-1}^-, Q_{n-1}^+]$............$[Q_1^-, Q_1^+]$	$[Q_0^-, Q_0^+]$	
s_1^n 0	n$[Q_n^-, Q_n^+]$	(n-1) $[Q_{n-1}^-, Q_{n-1}^+]$.........$[Q_1^-, Q_1^+]$		
s^{n-1}	$\frac{[Q_{n-1}^-, Q_{n-1}^+]}{n}$	$\frac{2[Q_{n-2}^-, Q_{n-2}^+]}{n}$..........	$[Q_0^-, Q_0^+]$	0

repeat he above procedure to get the numerator of reduced order s^k.

From the above two arrays we get our desired approximate lower order model of order k.

So, reduced order model will be,

$$R_k(s) = \frac{N_{Dk-1}(s)}{D_{Dk}(s)} \quad (4)$$

Where $N_{Dk-1}(s)$ and $D_{Dk}(s)$ represent the reduced numerator and denominator of reduced order obtained by differentiation method.

B. Routh stability criterion Method

Let the higher order transfer higher order system is,

$$H(S) = \frac{b_{11}s^m + b_{21}s^{m-1} + b_{12}s^{m-2} + b_{22}s^{m-3} + \cdots\ldots}{a_{11}s^n + a_{21}s^{n-1} + a_{12}s^{n-2} + a_{22}s^{n-3} + \cdots\ldots} \quad (3)$$

Here m<=n

Table III

Numerator Stability Array

b_{11}	b_{12}	b_{13}	b_{14} . . . . .
b_{21}	b_{22}	b_{23}	b_{24} . . . . .
b_{31}	b_{32}	b_{33}	. . . .
b_{41}	b_{42}	b_{43}	. . . .
.	.	.	.
.	.	.	.
.	.	.	.
$b_{m,1}$			
$b_{m+1,1}$			

Table IV

Denominator Stability Array

a_{11}	a_{12}	a_{13}	a_{14} . . . . .
a_{21}	a_{22}	a_{23}	a_{24} . . . . .
a_{31}	a_{32}	a_{33}	. . . .
a_{41}	a_{42}	a_{43}	. . . .
.	.	.	.
.	.	.	.
.	.	.	.
$a_{n-2,1}$	$a_{n-2,2}$		
$a_{n-1,1}$	$a_{n-1,2}$		
$a_{n,1}$			
$a_{n+1,1}$			

The above two tables III and IV represents routh stability array for the numerator and denominator polynomial respectively.

Now from the above two tables we get the required reduced order model which is represented as follows:

$$R_k(s) = \frac{N_{Rk-1}(s)}{D_{Rk}(s)} \qquad (5)$$

Where $N_{Rk-1}(s)$ and $D_{Rk}(s)$ represent the reduced numerator and denominator of reduced order obtained by routh stability criterion method.

C. Mixed Methods

CASE 1:- Determination of the numerator and denominator polynomial of the K^{th} order reduced model by differentiation method and routh stability criterion method respectively. So, the reduced order model is represented by

$$R_{DRk}(s) = \frac{N_{Dk-1}(s)}{D_{Rk}(s)} \qquad (6)$$

CASE 2:- Determination of the numerator and denominator polynomial of the K^{th} order reduced model by routh stability criterion method and differentiation method respectively. So, the reduced order model is represented by

$$R_{RDk}(s) = \frac{N_{Rk-1}(s)}{D_{Dk}(s)} \qquad (7)$$

4. NUMERICAL EXAMPLE

Let the stable seventh order Interval system considered be defined as[1,17]

$$G(s) = \frac{[1.9,2.1]s^6 + [24.7,27.3]s^5 + [157.7,174.3]s^4 + [542,599]s^3 + [930,1028]s^2 + [721.8,797.8]s + [187.1,206.7]}{[0.95,1.05]s^7 + [8.799,9.703]s^6 + [52.23,57.33]s^5 + [182.9,202.1]s^4 + [429,474.2]s^3 + [572.5,632.7]s^2 + [325.3,359.5]s + [57.35,63.39]}$$

Method 1: Reduction of both numerator and denominator by differentiation method.

So from seventh order model to get second order model we need to differentiate is n-k = 7-2 = 5.

The numerator and denominator array according table I and II are as follows

Numerator Array

S^6	1.9, 2.1	24.7, 27.3	157.7, 174.3	542, 599	930, 1028	721.8, 797.8	187.1, 206.7
S^5	4.117, 4.55	52.27, 58.1	271, 299.5	620, 685.3	601.5, 664.8	187.1, 206.7	
S^4	10.51, 11.62	108.4, 119.8	372, 411.2	481.2, 531.9	187.1, 206.7		
S^3	27.1, 29.95	186, 205.6	360.9, 398.8	187.1, 206.7			
S^2	62, 68.53	240.6, 265.9	187.1, 206.7				
S^1	120.3, 132.9	187.1, 206.7					
S^0	187.1, 206.7						

So from the numerator array reduced numerator polynomial of 2^{nd} Order model is,

$$N_D(s) = [120.3,132.9]s + [187.1,206.7]$$

Denominator Array

S^7	0.95, 1.05	8.779, 9.703	52.23, 57.73	182.9, 202.1	429, 474.2	572.5, 632.7	325.3, 359.5	57.35, 63.39
S^6	1.254, 1.386	14.92, 16.49	78.38, 86.61	245.1, 270.9	408.9, 451.9	278.8, 308.1	57.35, 63.39	
S^5	2.487, 2.749	26.13, 28.87	122.6, 135.5	272.6, 301.3	232.3, 256.8	57.35, 63.39		
S^4	5.225, 5.774	49.03, 54.2	163.6, 180.8	185.9, 205.4	57.35, 63.39			
S^3	12.26, 13.55	81.78, 90.38	139.4, 154.1	57.35, 63.39				
S^2	27.26, 30.13	92.94, 102.7	57.35, 63.39					
S^1	46.47, 51.36	57.35, 63.39						
S^0	57.35, 63.39							

So from the denominator array reduced denominator polynomial of 2^{nd} Order model is,

$$D_D(s) = [27.26,30.13]s^2 + [92.94,102.7]s + [57.35,63.39]$$

So, the 2^{nd} order reduced model obtained by differentiation method be like,

$$R_{DD}(s) = \frac{N_D(s)}{D_D(s)} = \frac{[120.3,132.9]s + [187.1,206.7]}{[27.26,30.13]s^2 + [92.94,102.7]s + [57.35,63.39]}$$

The step response for original seventh order and reduced second order model obtained by differentiation method for lower bound and upper bound are shown in figures 1 and 2 respectively,

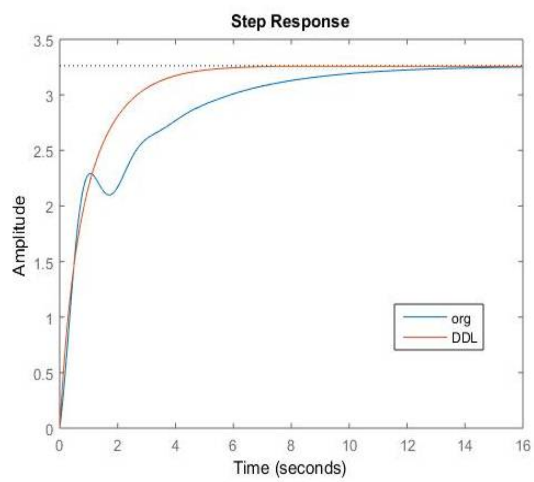

Figure 1: Step response of original model and ROM obtained by differentiation method for lower bound.

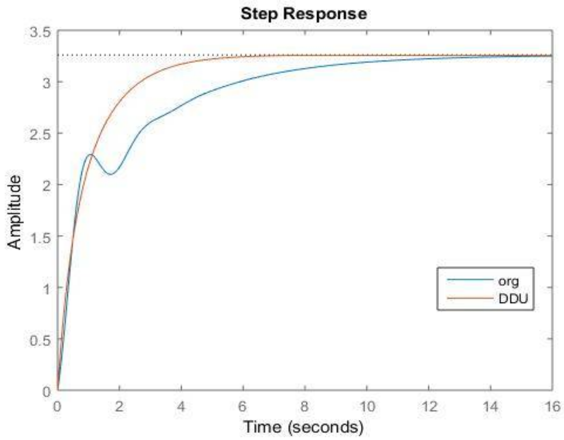

Figure 2: Step response of original model and ROM obtained by differentiation method for upper bound.

Method 2: Reduction of both numerator and denominator by routh stability criterion method.

The numerator and denominator routh array table for seventh order model is obtained according to table III and IV described above in this paper.

So the numerator and denominator stability array for seventh order model is,

Numerator Stability Array

S^6	1.9, 2.1	157.7, 174.3	930, 1028	187.1, 206.7
S^5	24.7, 27.3	542, 599	721.8, 797.8	
S^4	116.01, 128.22	874.47, 966.63	187.1, 206.7	
S^3	355.81, 393.19	681.96, 753.79		
S^2	652.13, 720.81	187.1, 206.7		
S^1	579.88, 641.039			
S^0	187.1, 206.7			

From this Numerator stability array which is obtained by routh approximation method we can form reduced order numerator polynomial of any order less than the order original system so, reduced 2nd order numerator polynomial of the given model is written as,

$$N_{R1}(s) = [579.88, 641.039]s + [187.1, 206.7]$$

Similarly, we can obtained reduced 2nd order denominator polynomial from the Denominator stability array which is also obtained by routh approximation method.

So, the required denominator polynomial is,

$$D_{R2} = [347.05, 383.59]s^2 + [267.36, 295.46]s + [57.35, 63.39]$$

Denominator Stability Array

S^7	0.95, 1.05	52.23, 57.73	429, 474.2	325.3, 359.5
S^6	8.779, 9.703	182.9, 202.1	572.5, 632.7	57.35, 63.39
S^5	32.437, 35.86	367.05, 405.73	319.09, 352.64	
S^4	83.559, 92.316	486.14, 537.28	57.35, 63.39	
S^3	178.33, 197.03	296.83, 328.02		
S^2	347.05, 383.59	57.35, 63.39		
S^1	267.36, 295.46			
S^0	57.35, 63.39			

So, the 2nd order reduced model obtained by routh stability criterion method is,

$$R_{RR}(s) = \frac{N_{R1}(s)}{D_{R2}(s)} = \frac{[579.88, 641.039]s + [187.1, 206.7]}{[347.05, 383.59]s^2 + [267.36, 295.46]s + [57.35, 63.39]}$$

The step response for original seventh order and reduced second order model obtained by routh stability criterion method for lower bound and upper bound are shown in figures 3 and 4 respectively.

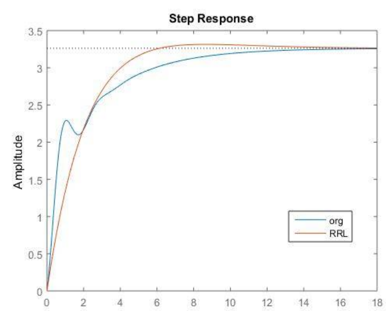

Figure 3: Step response of original model and ROM obtained by routh stability criterion method for lower bound.

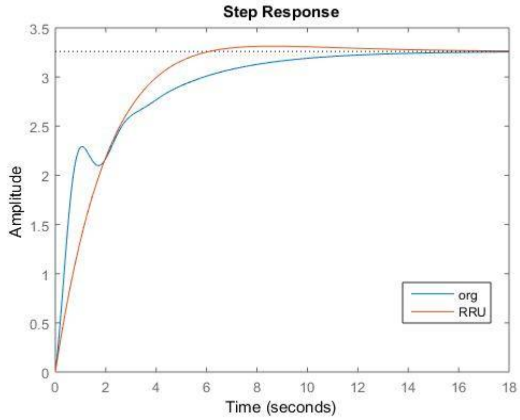

Figure 4: Step response of original model and ROM obtained by routh stability criterion method for upper bound.

Method 3: Mixed Methods

Case I: Reduction of numerator for reduced order model by differentiation method and reduction of denominator for the same by routh stability criterion method.

So the required 2nd order reduced model is,

$$R_{DR}(s) = \frac{N_{DR1}(s)}{D_{R2}(s)} = \frac{[120.3,132.9]S + [187.1,206.7]}{[347.05,383.59]s^2 + [267.36,295.46]s + [57.35,63.39]}$$

Step response of model obtained by case I with original model is shown below in figure 5 and 6.

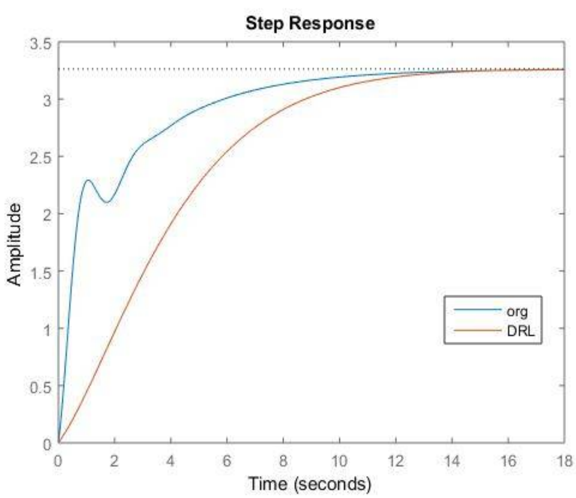

Figure 5: Step response of original model and ROM obtained by mixed method(numerator by differentiation and denominator by routh approximation) for lower bound.

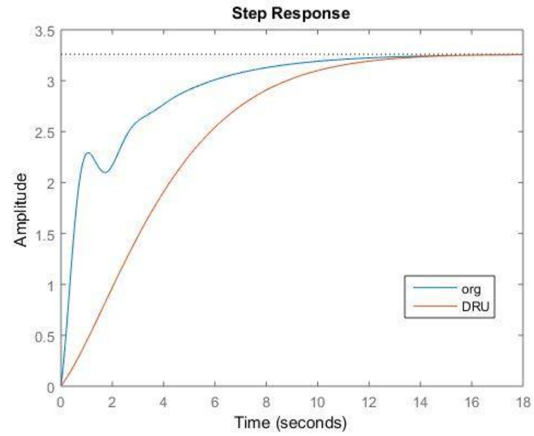

Figure 6: Step response of original model and ROM obtained by mixed method(numerator by differentiation and denominator by routh approximation) for upper bound.

Case II: Reduction of numerator for reduced order model by routh stability criterion method and reduction of denominator for the same by differentiation method.

So reduced model is,

$$R_{RD}(s) = \frac{N_{R1}(s)}{D_{D2}(s)} = \frac{[579.88,641.039]s + [187.1,206.7]}{[27.26,30.13]s^2 + [92.94,102.7]s + [57.35,63.39]}$$

Step response of model obtained by case I with original model is shown below in figure 7 and 8.

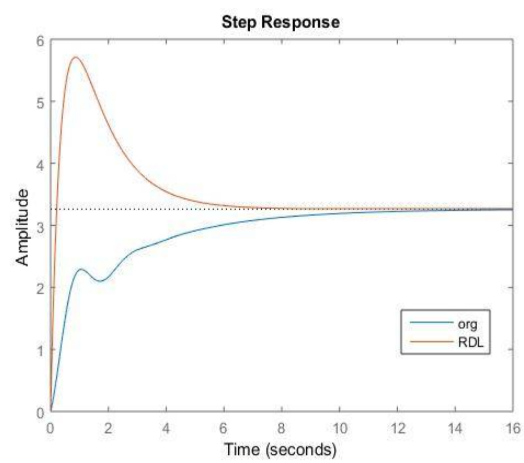

Figure 7: Step response of original model and ROM obtained by mixed method(numerator by routh approximation and denominator by differentiation) for lower bound.

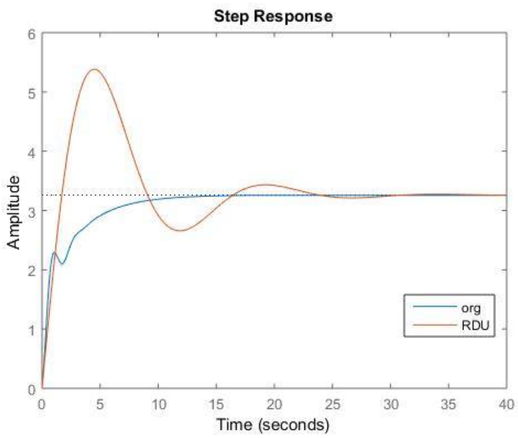

Figure 8: Step response of original model and ROM obtained by mixed method(numerator by routh approximation and denominator by differentiation) for upper bound.

The 2nd order reduced model obtained by the kharitonov's theorem method[13] is given as

$$R_{KK2}(s) = \frac{[1.3146,1.6057]s + [0.3406,0.416]}{[1,1]s^2 + [0.5924,0.7235]s + [0.1044,0.1275]}$$

Step response for above reduced model obtained by kharitonov's theorem method for lower and upper bound is shown in figure 9 and 10 respectively.

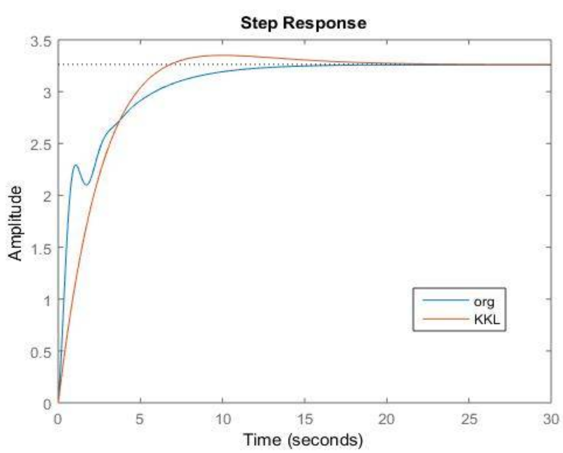

Figure 9: Step response of original model and ROM obtained by kharitonov's theorem method for lower bound.

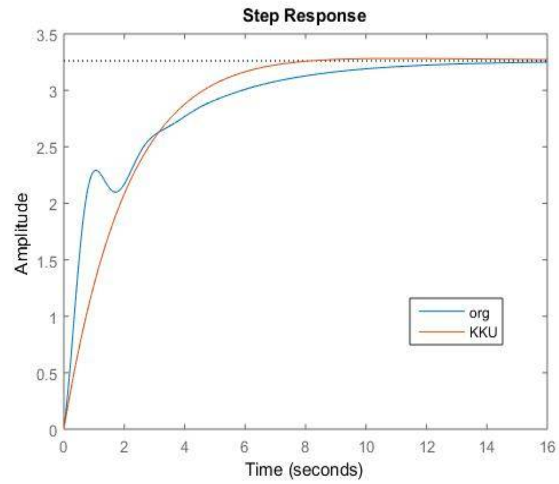

Figure 10: Step response of original model and ROM obtained by kharitonov's theorem method for upper bound.

5. CONCLUSION

This paper proposed mixed methods for reduction of higher order model to reduced order model. In mixed method two different methods are used for finding parameters of the numerator and denominator. Here Differentiation method and Routh stability criterion are used and then result is compare with reduced model obtained by Kharitonov's theorem and Routh Approximation and it is observed that model obtained from proposed method is more close to original model in comparison with model obtained from kharitonov's theorem method and it is also observed that proposed method is mathematically much easier then other methods and it gives all possible lower order model for given higher order model.

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